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Selecting Appropriate Dimension Stone in Processing Plants using the Fuzzy Delphi Analytic Hierarchy Process

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ABSTRACT

Selecting the most appropriate dimension stones has always been one of the vital issues in processing plants which stone is processing. This study aims to develop a new hierarchical model to evaluate and rank the dimension stone with the use of effective and major criteria such as beauty and durability, stone slab ability, supply chain, profitability, domestic and foreign market demand, and environmental factors and simultaneously, considering subjective judgments of decision-makers. The proposed approach is based on the fuzzy Delphi analytic hierarchy process method to calculate the effectiveness of the criteria and rank the dimension stone by decision-makers. This model is utilized for some stone process plants to assess and rank some famous Iranian granite. Then, the most appropriate stones for stone processing process plants are ranked. The ranking results showed that Chayan black granite with a 0.24 value, is the best stone to sawing in studied stone process plants. Finally, it was deduced that the most appropriate selection of dimension stone can reliably be done using the developed approach according to the most important effective criteria.

1. Introduction

Dimension stone can be defined as a natural rock material that is quarried to obtain blocks or slabs that have specific sizes and shapes [1,2]. The main use of dimension stone is within the building industries, which makes up more than 80% of what is used [3]. Another use of dimension stones is in preparing funeral monuments industry accounting for 15%, and other various special applications for around 5% [4]. Iran is a country in the southwest of Asia and the Middle East, which has very great potential in the production and export of dimension stones.

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Nevertheless, in the last 10 years, in spite of growing mine production in many cases, this potential has been overlooked. In spite of that Iran's dimension stone products have high quality by considering criteria like structural characteristics and decorative in the world, but it has taken a small share of the global markets of dimension stones. In terms of diversity of quality, texture, color, and economic factors, some of these reserves are unique and these reserves have high worth from the view of job creation and income generation [5]. On the other hand, the important factor in the market of dimension stones is their quality at the output of stone-cutting process plants. If naturally high-quality dimension stones are not cut, worn, and polished well in process plants, they will not attract the attention of any customer because they will be not attractive and pretty. The quality of dimension stone production depends on the economic justification of the production process.

One of the most effective factors in the economy of dimension stone-cutting process plants is selecting the appropriate dimension stone according to the characteristics of the cutting, subbing, and polishing equipment of that factory [6,7]. If the type of dimension stone is not selected correctly, there will be many problems for the equipment in the different steps of preparing the dimension stone, and in addition to the depreciation devices, it will also change how good the product is in the end. In addition, the cutting and preparation of unsuitable dimension stones for the equipment factory increases energy consumption and consequently increases the costs of the factory [8,9]. According to the above-mentioned reasons, selecting the right stone for each factory has a significant impact on the final quality of the dimension stone produced, and for this reason, this subject has been the focus of various researchers [10,11]. In the following, a number of related researches are discussed.

This research aims to develop a new hierarchical model to evaluate and rank stone dimensions using effective and major criteria such as beauty and durability, stone slab capability, supply chain, profitability, domestic and foreign market demand, and environmental issues, while subjective judgments of decision makers are being seen.

The suggested methodology relies on the fuzzy Delphi analytic hierarchy process (FDAHP) technique to rank the dimension stone by decision-makers. This model is applied to some of Urmia's stone process plants to evaluate and rank some famous Iranian granite including Yazd red granite, Nehbandan white granite, Chayan black granite, Khoramdareh chocolate granite, and Mashhad pearl granite. Then, the most appropriate stones from the viewpoint of the mentioned criteria for stone processing factories are ranked.

This research presents several key contributions aimed at improving the selection process of dimension stones in stone processing plants. The main contributions are listed and explained in detail below:

i. *Development of a hierarchical model for dimension stone selection* – This study introduces a structured decision-making framework using FDAHP. The model integrates multiple technical, economic, and environmental factors to rank dimension stones effectively. By providing a systematic and objective evaluation method, this model reduces uncertainty in decision-making and ensures that stone processing plants can select the most suitable stones based on comprehensive criteria. This helps optimize production efficiency and profitability, leading to better resource utilization in the industry.
ii. *Incorporation of economic and market-based criteria* – Unlike traditional selection methods that focus only on physical and mechanical properties, this research includes profitability, domestic and international market demand, and supply chain stability as key decision factors. By integrating economic considerations, the study ensures that selected stones are not only technically viable but also financially beneficial. This approach helps

stone processing plants improve competitiveness and sustainability by aligning production with market demand and cost efficiency.

iii. *Quantitative evaluation of cutting costs and profitability* – The study develops new cost estimation models based on the rock sawability index, energy consumption, and tool wear rates. These models provide an accurate assessment of cutting costs and profitability for different stones. By offering precise cost-benefit analysis, this contribution helps manufacturers minimize production costs while maintaining quality standards. It also supports better financial planning and investment decisions, ensuring that stone processing plants maximize their return on investment.

iv. *Consideration of environmental and worker safety aspects* – The study incorporates environmental impact factors, particularly dust pollution and harmful mineral content released during the cutting process. This ensures a more sustainable and work-er-friendly approach to stone selection. By recognizing and addressing health and environmental risks, this research promotes safer working conditions and reduces the ecological footprint of stone processing. This contribution is particularly important as global industries move towards more sustainable and environmentally responsible practices.

2. Literature Review

Many studies have been conducted on the ranking and classification of dimension stones used for buildings based on how easily they can be cut and some other strength features [12-14], but it can be said that no research has been done on ranking based on profit function and cutting costs so far.

Mikaeil *et al.* [15] conducted a ranking of dimension stones based on their sawability utilizing the PROMETHEE technique. This research focused on four key mechanical and physical properties of stones: Mohs hardness, Young's modulus, and uniaxial compressive strength, which were established as the evaluation criteria. Consequently, a decision support system was created to facilitate the ranking of dimension stone sawability. In this investigation, both the FDAHP and PROMETHEE methods were employed to assess sawability. The FDAHP method was utilized to ascertain the weights of the criteria, while the PROMETHEE method was applied for the actual ranking of sawability. This innovative ranking approach can be implemented to evaluate the production rate of dimension stones across various stone manufacturing facilities with different types of rocks.

Mehdi Hosseini *et al.* [16] studied the effect of the cooling and lubricant fluid on the cutting performance of dimension stones through artificial intelligence models. The modeling was conducted utilizing two approaches: artificial neural networks (ANN) and a hybrid genetic algorithm combined with an artificial neural network; i.e. hybrid GA-ANN algorithm. The input data comprised four rock parameters along with two operational parameters, which were the depth of cut and feed rate. The output data represented the maximum electrical current. According to the results, 18 models were produced. Among them, the multi-layer perceptron (MLP) neural network was better at making accurate and reliable predictions than the combined ANN-GA method. The suggested MLP models show that using soap water mixed in a ratio of 1 part soap to 40 parts water can help improve cutting performance while using less electricity.

Afradi *et al.* [17] applied a method for quarrying dimension stones in order to increase productivity and production. In their study, conventional methods of dimension stone quarrying were used. They looked at how to cut down on waste and save time in stone mining by using a method that considers different factors, like making more money, safety, quality, and less harm to the environment. The results showed that using diamond wire cutting is the best way to extract large stones in the areas that are looked at.

Mikaeil *et al.* [18] worked on a clear way to predict how much energy-cutting machines use, based on statistics. They did lab tests in various working conditions. They looked at 12 samples of soft and hard rocks in a lab using a cutting machine. Then, the samples were taken to the rock mechanics lab to find out Schimazek's F-abrasiveness factor. After measuring how much the samples wear down, they did some statistical analysis and introduced new methods to predict how much the cutting machine will be used based on the wear and the depth of the cut. The recommended statistical models can be used confidently to measure the current used during the cutting process.

Shaffiee Haghshenas *et al.* [19] conducted an assessment in 2022 with respect to the adequacy of cutting machines utilized within the measurement stone cutting handle through factual investigation. At first, research facility tests were carried out beneath different operational conditions, analyzing distinctive cutting profundities and bolster rates over 12 tests of both delicate and difficult rocks. In this way, the mechanical properties of these tests were evaluated in a shake mechanics research facility. The factual examination was executed utilizing the SPSS program, centering on cutting depth, the mechanical properties of the rock samples, and feed rate to estimate the electrical current utilization of the cutting machine. The discoveries demonstrated that the proposed measurable strategy is exceedingly dependable for evaluating the electrical current utilized amid the cutting handle.

Hayati *et al.* [20] proposed an improved multi-attribute decision-making-based SWOT analysis for strategic planning in the dimension stones industry. Three main ideas were recommended: setting up a stone research center, hiring expert workers, and promoting the brand through online sales. The identified strategies are proper for keeping this from happening to environmental damage, making strides in financial development, and paying consideration to social issues.

Bagloo & Ataee-pour [21] classified measurement stones utilizing AHP and fluffy rationale. In their classification framework, the values were categorized as takes after: the powerless bunch ranged from 1 to 2, the direct bunch ranged from 2 to 2.5, and the great bunch was characterized as extending from 2.5 to 3. In their work, the SQP show was utilized. It was demonstrated that its scoring method stayed steady for different sorts of stones.

3. Field and Laboratory Studies

In this research, five common types of dimension stone from Iranian quarries including red granite (A_1), black granite (A_2), white granite (A_3), Khoramdeh chocolate granite (A_4), and pearl granite (A_5) were sampled. Then, they were transferred to the laboratory for testing and data acquiring. The locations of studied quarries and stone processing plants are shown in Figure 1.

For physical and mechanical tests, sample blocks were prepared. Regular tests were conducted to check the four main physical and mechanical qualities of stones which include uniaxial compressive strength (UCS), Mohs hardness (MH), Young’s modulus (YM), and Schmiazek abrasiveness factor (SF-a), that were used to assessing dimension stone sawability.

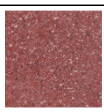
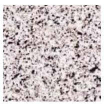
Fig. 1. Location of studied quarries on Iran map

The results of laboratory tests are given in Table 1. It should be mentioned that the values in Table 1 are average quantities that are obtained from statistical analysis that was done on a group of data that was acquired from laboratory tests. Schimazek's F-abrasiveness factor is a dependent parameter that is calculated using other variables. In order to calculate Schimazek's F-abrasiveness factor, Eq. (1) is used:

$$\boldsymbol{SFa} = \frac{\boldsymbol{EQC} \times \boldsymbol{Gs} \times \boldsymbol{BTS}}{\mathbf{100}}, \tag{1}$$

where *EQC* is the equivalent quartz content percentage, *SFa* is the Schimazek's wear factor (N/mm), *BTS* is the indirect Brazilian tensile strength, and *Gs* is the mean grain size (mm).

Table 1
Result of laboratory tests of samples collected from quarries

Dimension stone sample		Commercial name	Name of quarry	BTS	EQ_C	Gs	SF-a	UCS	YM	MH
				MPa	%	mm	N/mm	MPa	GPa	n
A_1		Granite	Ghale Khargoshi	8.52	57.65	2.9	14.24	142	43.6	6.1
A_2		Granite	Chayan	15	60.06	0.87	7.6	173	48.6	6.6
A_3		Granite	Nehbandan	9.2	64.3	4.1	24.25	145	35.5	5.95
A_4		Granite	Khoshtinat	8.3	32.2	3.9	10.42	133	28.9	5.65
A_5		Granite	Khatam	7.4	30.3	3.8	8.5	125	31.2	5.6

4. Main Criteria for Ranking Dimension Stones

There are different parameters for ranking decorative stones. Each of these parameters represents different aspects of stone properties. Some parameters have been used many times in different research works. But on the other hand, some other parameters, while they are important, have not been taken into consideration. In this research work, which was focused mostly on the economic aspects of the cutting process, we tried to use parameters related to economic issues. In the following, the criteria used to rank decorative stones are introduced and explanations are given in connection with each one.

4.1 Beauty and Durability (C_1)

The use of stones as a cover for buildings and urban areas has grown. Beauty and durability are some of the most important criteria in decision-making and their selection. The place where the stones are used depends on the rock strength parameters. The stones used in the exterior facing of a building should be resistant to climatic factors such as acidic rains, sunlight, freezing, etc. But for the stones that are used for indoor facing of buildings, bearing pressure and abrasive resistance are more important. Hence, in the selection process, the hardness of the rock and the stone's compressive must be considered. Beauty by itself is a qualitative measure and depends on the different tastes of people. Generally, in the market, the stones that have brighter colors are so popular and market-friendly.

4.2 Domestic Market Demand (C_2)

Domestic market demand is one of the important criteria for processing planning because of creating income and profits for stone processing plants. Processing planning, besides the other criteria, tries to select the optimal option (stone) by considering the domestic market demand. After profitability, domestic market demand can be considered as the most important criterion in selecting the dimension stone. Processing planning can increase its income by increasing its sales, which can be possible by having more requests and demands for stone in the domestic market.

4.3 Foreign Market Demand (C_3)

Nowadays, in the global economy, the importance of production and export and their effects on countries' economic status is so obvious. The development of non-oil exports can help the countries in many cases like an economic exchange and creating jobs. Iran has a great potential to attend in global markets due to its reserves of dimensional stones with a variety of types. Because of some problems in domestic markets such as lack of liquidity, manufacturers tend to be present in the foreign market. On the other hand, the government supported the manufacturers to be in international markets because of the economic exchange and lack of liquidity.

4.4 Environmental Issues (C_4)

Environmental criterion besides the other criteria has a key role in selecting the optimal option for the stone processing plants. Pollution inside the factory, which is produced from the dust of the cutting process, is one of the environmental challenges that can be too harmful to workers' health. So, stone processing plants should pay attention to their selections. In this study, the environmental criterion has been studied from the viewpoint of the percentage of harmful minerals such as silica. So, if the percentage of these minerals is high, it can make harm to workers during the cutting process.

4.5 Supply Chain (C_5)

In process planning, various criteria affect the selection of the stones. Process planning selects and buys the stones according to these criteria. One of the important criteria in selecting the optimal stone is the supply chain. For further explanation about this criterion consider that a type of stone may have a high beauty and cutting ability, but supplier mine is not able to provide the ordered tonnage which should be supplied for market demand. Therefore, manufacturers always try to select the stone that has a stable and flexible supply chain.

4.6 Block Quality-Slab Efficiency (C_6)

At the moment, one of the important parameters to be considered in the cutting industry is block quality. The block quality can be defined as the slab efficiency of each block. The slab efficiency is the number of intact slabs divided by the number of slabs that are ideally and theoretically obtained from the block. Generally, the block quality is divided into qualitative categories by expert laborers in stone processing plants.

4.7 Profitability Index (C_7)

The profitability index is the most important criterion for selecting the optimal stone from the viewpoint of processing planning. The profitability index is a tool that looks at how the costs compare to the benefits of a project. It helps compare projects by showing how much value is made for each dollar spent.

For calculating the amount of profitability, the cutting costs should be noted. These indexes are defined by Eq. (2) and Eq. (3). In this way, the profitability of each stone can be provided by two presented models, which are used in the cutting process for calculating cutting cost per meter-cubic:

$$\boldsymbol{I_G = 10^{2.098} \times \frac{D_c{}^{0.418} \times F_r{}^{0.118}}{RSi^{0.247}}}, \tag{2}$$

$$\boldsymbol{W_r = \frac{D_c{}^{1.166} \times F_r{}^{1.173}}{10^{0.564} \times RSi^{1.372}}}, \tag{3}$$

where I_G is the indicant of current consumption for cutting granite stone in terms of the ampere, W_r is the rate of abrasion in terms of mm per hour, D_c is the depth of cutting in terms of mm, F_r is the rate of progress in terms of cm per second, and RSi is the rock sawability index.

According to Table 2, the rock sawability index is calculated according to the mechanical properties of stone, including UCS, SFa, MH, and YM for two groups of soft and hard stones.

In Eq. (4) and Eq. (5), C_{EG} is the indicant of the cost of consumed energy for hard stones in terms of dollars (\$) per square meter, C_d is the indicant of the used discs in terms of dollars (\$) per square meter, and V is the indicant of the consumed voltage of the cutting device:

$$\boldsymbol{C_{EG} = \frac{10^{2.098} \times D_c{}^{0.418} \times F_r{}^{0.118} \times V \times E_P}{RS_i{}^{0.247} \times D_c \times F_r \times K}}, \tag{4}$$

$$\boldsymbol{C_d = \frac{D_c{}^{1.166} \times F_r{}^{1.173} \times (P_p + (n \times (S_p + G_p)))}{10^{0.564} \times RS_i{}^{1.372} \times L \times n \times D_c \times F_r \times K}}. \tag{5}$$

Table 2
Rock sawability index reference table

	Parameters		Value				
1	UCS (MPa)	soft	25>	25-55	55-85	85-115	115<
		hard	50>	50-90	90-140	140-200	200<
			very low strength	low strength	average strength	high strength	very high strength
	Score		37	25.9	18.5	9.25	3.7
2	SF_a (N/mm)	soft	0.05>	0.05-0.1	0.1-0.15	0.15-0.2	0.2<
		hard	2.5>	2.5-5	5-7.5	7.5-10	10<
			abrasiveness very low	abrasiveness low	average abrasiveness	abrasiveness high	abrasiveness very high
	Score		36	25.2	18	9	3.6
3	MH (n)	soft	2>	2-2.5	2.5-3	3-3.5	>3.5
		hard	3>	3-4.5	4.5-6	6-7	>7
			very soft-soft	nearly soft	nearly hard	hard	very hard
	Score		17	11.9	8.5	4.25	1.7
4	YM (GPa)	soft	10>	10-30	30-50	50-70	70<
		hard	15>	15-40	40-65	65-90	90<
			very low	low	average	high	very high
	Score		10	7	5	2.5	1

The function of overall costs for hard stones can be rewritten like in Eq. (6):

$$\boldsymbol{C_{TG}} = \frac{\boldsymbol{D_c}^{1.166} \times \boldsymbol{F_r}^{1.173} \times (\boldsymbol{P_p} + (\boldsymbol{n} \times (\boldsymbol{S_p} + \boldsymbol{G_p})))}{10^{0.564} \times \boldsymbol{RSi}^{1.372} \times \boldsymbol{L} \times \boldsymbol{n} \times \boldsymbol{D_c} \times \boldsymbol{F_r} \times \boldsymbol{K}} + \frac{10^{2.098} \times \boldsymbol{D_c}^{0.418} \times \boldsymbol{F_r}^{0.118} \times \boldsymbol{V} \times \boldsymbol{E_P}}{\boldsymbol{RSi}^{0.247} \times \boldsymbol{D_c} \times \boldsymbol{F_r} \times \boldsymbol{K} \times 1000} + \frac{\boldsymbol{P} \times \boldsymbol{N_w}}{\boldsymbol{H_m} \times \boldsymbol{D_c} \times \boldsymbol{F_r} \times \boldsymbol{K}} + \frac{\boldsymbol{M_p}}{\boldsymbol{D_c} \times \boldsymbol{F_r} \times \boldsymbol{K} \times \boldsymbol{d_y} \times \boldsymbol{H_d}} + \frac{\boldsymbol{W_{co}} \times \boldsymbol{W_P}}{\boldsymbol{D_c} \times \boldsymbol{F_r} \times \boldsymbol{K}}, \tag{6}$$

where C_{TG} is the overall production costs for hard stones in terms of dollars per square meter. By using Eq. (6), stone sawability could be calculated based on variables that play an important role in the stone-cutting process. For this purpose, cutting tests for different hard stones (stones with rock sawability index ranges 15 to 90) under constant operating conditions (with a depth of cutting of 0.5 mm and a progress rate of 60 cm/s) were done. The profit is defined as Eq. (7):

$$\boldsymbol{B} = \boldsymbol{S_P} - \boldsymbol{C_{TG}} - \frac{1000}{(\boldsymbol{T_p} + \boldsymbol{T_s}) \times \boldsymbol{\gamma}}, \tag{7}$$

where γ is the density of stones (kg/m3), T_p is the cutting stone plaque thickness (m), T_s is the thickness of a segment, and S_P is the price of the cutting stone per surface unit. It should be noted that dimension stone cubes are ideal and there are no discontinuities or fractions in the cube's body.

5. Dimension Stone Ranking

Traditional methods of multi-criteria decision-making, despite their capabilities in solving problems related to this field, do not provide acceptable performance when faced with a group of problems raised in different fields of engineering [22,23]. One of the important characteristics of these challenging problems is the qualitative descriptions of the conditions governing the problem, which create problems for use in mathematical calculations. To solve this problem, the use of methods based on fuzzy theory was suggested [24]. A variety of fuzzy decision-making methods were presented on this basis [25,26]. Among the various fuzzy methods, the FDAHP as a combined method is considered one of the most powerful methods in carrying out different steps of weighting different criteria and finally ranking them. Therefore, in this research work, considering the capabilities of the

FDAHP method, this method was used to rank dimension stones. In Figure 2, a flowchart of the steps taken to perform the ranking process is shown.

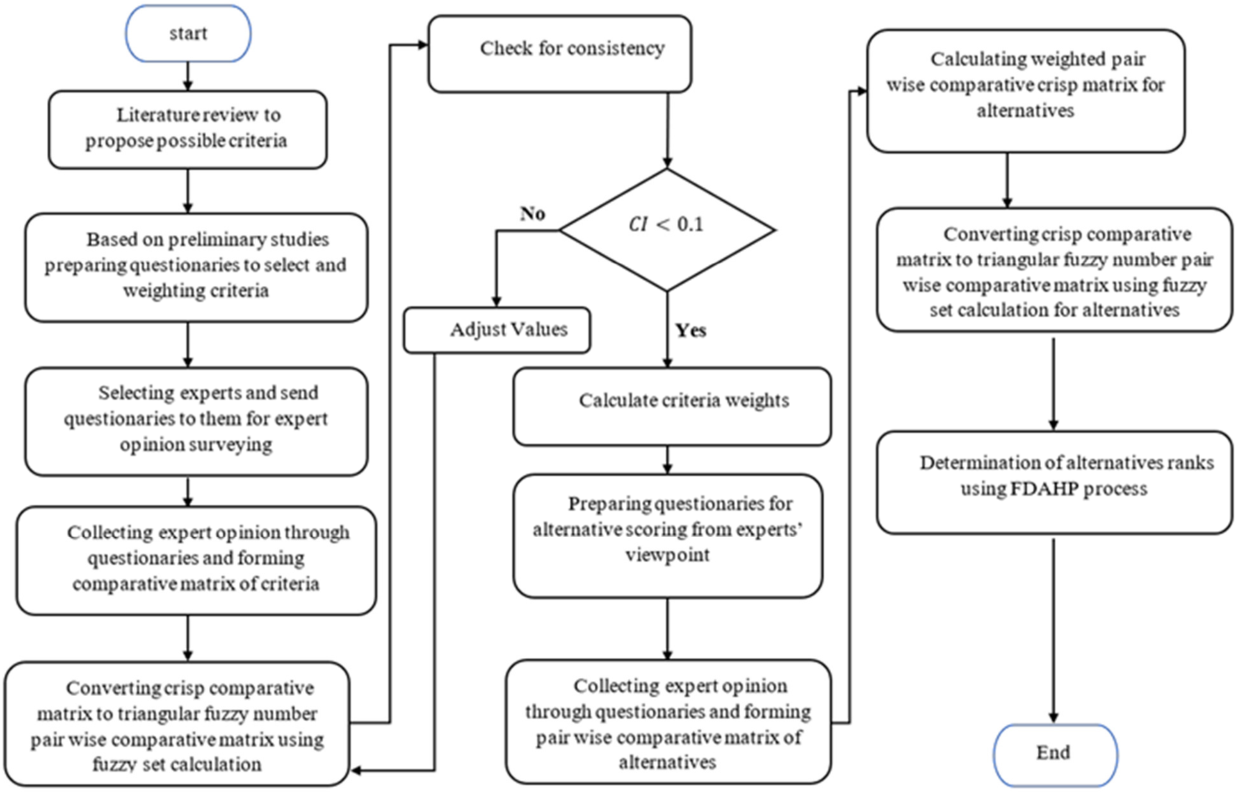


Fig. 2. Flowchart of various steps of dimension stones ranking process

5.1 Determination of Criterion Weights

The first step in the ranking process is to assign weight to each criterion. Criteria can be weighted using different methods. In fact, the weight of each criterion shows the effectiveness of each criterion in the ranking process. Therefore, the correct allocation of weight to each of the criteria is considered one of the most key steps. In this case, to create the comparison matrix, the fuzzy Delphi method (FDM) was applied. To achieve this goal, first of all, the technical questionnaires were prepared based on the Saaty rating scale and sent to experts [27]. After receiving experts' responses, according to the obtained results, a comparison matrix was formed.

In this research, C_1, C_2, …, C_7 symbolize the criteria, while aij demonstrates a quantified judgment on a couple of elements like C_i and C_j. The relative significance of the pair of elements comes from the division of C_i by C_j, which is based on the questionnaire. So, *Ä* matrix (7 × 7 matrix) was obtained based on above mentioned descriptions [28]. Eq. (8) shows the structure of the paired comparison matrix:

$$\ddot{A} = [\boldsymbol{a_{ij}}] = \begin{bmatrix} \mathbf{1} & \boldsymbol{a_{12}} & \cdots & \boldsymbol{a_{17}} \\ \mathbf{1}/\boldsymbol{a_{21}} & \mathbf{1} & \cdots & \boldsymbol{a_{27}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{1}/\boldsymbol{a_{71}} & \mathbf{1}/\boldsymbol{a_{72}} & \cdots & \mathbf{1} \end{bmatrix}. \tag{8}$$

Table 3 shows the prepared paired comparison matrix in line with experts' opinions.

Table 3
Comparison matrix of stones ranking

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
C_1	1	3	0.33	5	0.33	0.33	3
C_2	0.33	1	0.20	3	0.20	0.20	1
C_3	3	5	1	7	1	1	5
C_4	0.20	0.33	0.14	1	0.14	0.14	0.33
C_5	3	5	1	7	1	1	5
C_6	3	5	1	7	1	1	5
C_7	0.33	1	0.20	3	0.20	0.20	1

For forming the paired comparison matrix, it is essential to follow four steps [28]:

Step 1. In order to convert crisp numbers to triangular fuzzy numbers (TFN), the triangular membership function was used. TFN calculation membership function schematics are shown in Figure 3.

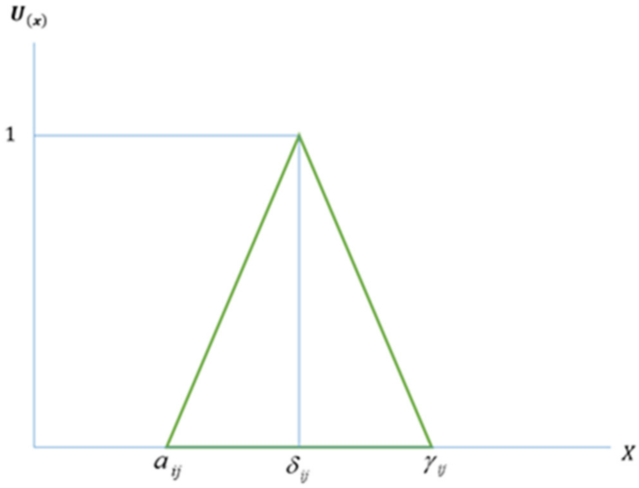


Fig. 3. Membership function graph of the TFN calculation by Delphi method

In the FDM, TFNs are calculated by using the Eqs. (9)-(12):

$$\tilde{a}_{ij} = (\alpha_{ij}, \delta_{ij}, \gamma_{ij}), \tag{9}$$

$$\alpha_{ij} = Min(\beta_{ijk}), k = 1, \dots, n, \tag{10}$$

$$\delta_{ij} = \left(\prod_{k=1}^{n} \beta_{ijk}\right)^{1/n}, k = 1, \dots, n, \tag{11}$$

$$\gamma_{ij} = Max(\beta_{ijk}), k = 1, \dots, n. \tag{12}$$

where a_{ij}, δ_{ij}, and γ_{ij}, are obtained by Eqs. (10)-(12). In above equations, a_{ij} demonstrates the lower bound and γ_{ij} is the upper bound. Also, β_{ijk} shows the relative importance of criterion *i* compared to *j* from k^{th} expert's opinion. It should be noticed that the number of experts was symbolized by k.

Step 2. The fuzzy pair-wise comparison matrix was formed by using the calculated FTN from the above steps. **Error! Reference source not found.** shows the prepared fuzzy pairwise comparison matrix for seven criteria.

Table 4
Sample of the prepared pairwise comparison matrix

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
C_1	(1,1,1)	(0.2,0.42,3)	(0.33,1.17,3)	(1,5.42,7)	(0.33,0.6,7)	(0.33,0.456,1)	(0.2,0.25,3)
C_2	(5,2.35,0.33)	(1,1,1)	(0.2,0.36,5)	(3,8.59,9)	(0.2,1.68,3)	(0.2,1.272,3)	(0.33,0.45,1)
C_3	(3,0.85,0.33)	(1,0.18,0.14)	(1,1,1)	(1,4.877,7)	(0.2,0.68,3)	(0.33,0.39,1)	(0.2,0.23,5)
C_4	(1,0.18,0.14)	(0.33,0.11,0.11)	(1,0.2,0.14)	(1,1,1)	(1,1.32,1)	(0.14,0.12,0.33)	(0.11,0.09,0.33)
C_5	(3,1.66,0.14)	(5,0.59,0.33)	(5,1.47,0.33)	(1,0.75,1)	(1,1,1)	(0.33,0.73,3)	(0.2,0.42,5)
C_6	(3,2.19,1)	(5,0.78,0.33)	(3,2.56,1)	(7,8.26,3)	(3,1.36,0.33)	(1,1,1)	(0.33,0.35,5)
C_7	(5,4,0.33)	(3,2.19,1)	(5,4.34,0.2)	(9.09,10.98,3)	(5,2.35,0.2)	(3,2.78,0.2)	(1,1,1)

Step 3. The relative fuzzy weight of the criteria is calculated based on Eq. (13) and Eq. (14):

$$\tilde{\boldsymbol{Z}}_{\boldsymbol{i}} = [\tilde{\boldsymbol{a}}_{\boldsymbol{i1}} \otimes \tilde{\boldsymbol{a}}_{\boldsymbol{i2}} \otimes \ldots \otimes \tilde{\boldsymbol{a}}_{\boldsymbol{in}}]^{\boldsymbol{1/n}}, \tag{13}$$

$$\widetilde{\boldsymbol{W}}_{\boldsymbol{i}} = \tilde{\boldsymbol{Z}}_{\boldsymbol{i}} \oslash (\tilde{\boldsymbol{Z}}_{\boldsymbol{i}} \oplus \cdots \oplus \tilde{\boldsymbol{Z}}_{\boldsymbol{n}}), \tag{14}$$

where $\otimes$, $\oplus$, and $\oslash$ denote the multiplication, summation, and division of the fuzzy numbers, respectively. Finally, $\widetilde{W}_i$ is a vector, that symbolizes the fuzzy weight of the i^{th} criterion.

Step 4. After figuring out the fuzzy weight for each criterion, we turn all the numbers into crisp numbers. This step is called the defuzzification process, and it uses Eq. (15) [29]:

$$\boldsymbol{W}_{\boldsymbol{i}} = \left(\prod_{\boldsymbol{i=1}}^{\boldsymbol{n}} \boldsymbol{\omega}_{\boldsymbol{i}}\right)^{\boldsymbol{1/n}}, \tag{15}$$

where W_i is the elements of the $\widetilde{W}_i$ vector. So, each weight that was calculated by using the FDAHP method is shown in Table 5.

Table 5
Calculated weight of the criteria by FDAHP

Criteria	Weights
C_1: Beauty and durability	0.07
C_2: Domestic market demand	0.14
C_3: Foreign market demand	0.06
C_4: Environmental issues	0.05
C_5: Supply chain	0.13
C_6: Stone slab stability	0.17
C_7: Profitability	0.38

5.2 Ranking the Dimension Stone Blocks

After determining the weight of each criterion, it is time to rank the stones. For ranking, the FAHP method is used. In fact, by using the weight of each criterion, by forming a comparative matrix and fuzzification, the priority of each alternative is determined. To select the best alternative according to the aforementioned criteria, all of the alternatives from the viewpoint of stone slab ability were assumed ideal, so the alternatives take equal scores. Based on the profit criteria, alternatives were compared and the best one was selected.

For evaluating and selecting the alternatives, some questionnaires which include questions about supply chain, foreign and domestic markets demand, beauty and durability, and environmental criteria were sent to experts. After receiving the responses of the experts through the collection of questionnaires, the comparative matrix of each criterion for each expert is formed and by multiplying it by the weight of each criterion, the comparative matrix of the weight of each criterion for each expert is obtained. Then, by summing all the weighted comparative matrices of each criterion of all experts, the overall weighted comparative matrix of each criterion is obtained. This matrix is converted into a weighted comparative matrix based on triangular fuzzy numbers using the described method and the ranking operation is performed. An example of an expert's prioritization for beauty and durability criteria based on the proposed Saaty scoring system is given in Table 6.

Table 6
Example of rated alternatives by experts considering beauty and durability criterion

Alternatives	Score
Chayan black granite	8
Yazd red granite	6
Nehbandan white granite	4
Khoramdareh chocolate granite	2
Mashhad pearl granite	1

Due to the acquired scores for alternatives according to the aforementioned criteria, the overall pairwise comparison matrix for the alternatives, which was prepared based on the experts' input, was obtained and rated by the FAHP method. As a sample, one of these alternative ratings is given in Table 8, which is according to the beauty and durability criterion (Table 7).

Table 7
Weight of alternatives considering beauty and durability criterion

Alternatives	Description	Weight
A_1	Chayan black granite	0.426
A_2	Yazd red granite	0.146
A_3	Nehbandan white granite	0.291
A_4	Khoramdareh chocolate granite	0.112
A_5	Mashhad pearl granite	0.025

Alternatives were rated according to other criteria like beauty and durability. In the final step, the alternatives' score was multiplied by the criteria weight, and the total scores for each option were obtained according to the criteria mentioned. Table 8, shows the final step, result, and rating of alternatives.

Considering Table 8, the first alternative (i.e. Chayan black granite) gains maximum scoring. Hence, based on the FDAHP method, this alternative was placed in the first position. In other words, taking into account the above criteria, Chayan black granite is the most suitable alternative for cutting and preparing for consumption in the market.

Table 8
Scores of the alternatives and criterion weights

Criteria weights	W_1	W_2	W_3	W_4	W_5	W_6	W_7	Rate
	0.07	0.14	0.06	0.05	0.13	0.17	0.38	
Alternatives	C_1	C_2	C_3	C_4	C_5	C_6	C_7	
A_1: Chayan black granite	0.42	0.16	0.17	0.23	0.28	0.2	0.24	0.24
A_2: Yazd red granite	0.14	0.05	0.1	0.38	0.15	0.2	0.34	0.23
A_3: Nehbandan white granite	0.29	0.3	0.42	0.23	0.19	0.2	0.15	0.22
A_4: Khoramdareh chocolate granite	0.11	0.25	0.2	0.07	0.21	0.2	0.14	0.17

A_5: Mashhad pearl granite	0.02	0.22	0.09	0.07	0.14	0.2	0.11	0.14

6. Conclusions

The existing variety of dimension stones and its huge resources in Iran, allow the stone-cutting plant managers to make decisions based on existing criteria and select the most suitable stone from the stones available in the market. This study aims to assess and select the best stone according to some important criteria in order to improve the stone-cutting process. All criteria were compared against each other to determine the degree of importance of each criterion and then all of them were rated based on the Saaty scale system from one to nine.

One of the major aspects of this research was designed to illustrate the capability of multiple criteria decision-making to rank and select the most appropriate dimension stones concerning some important criteria based on the opinions of experts. A natural stone should have certain criteria to be ranked as a dimension stone. These criteria are decorative, physical, and mechanical properties of stone and geological criteria. Additionally, in a stone factory, there are some technical criteria such as stone slab ability, supply chain, profitability, domestic and foreign market demand, and environmental issues that affect selecting the best stones for the sawing process. Five granites were selected as alternatives which are famous and popular in Iran. These alternatives were ranked using the FDAHP technique with respect to seven main criteria based on expert's opinion. Finally, Chayan black granite was suggested as the first alternative for sawing in stone processing plants. According to the results of this study, it seems that the proposed hierarchical model can be reliably used to evaluate and rank the dimension stones to make planning more efficient in stone processing plants.

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Conflicts of Interest

The author declares no conflicts of interest.

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