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An Aczel-Alsina T-Spherical Fuzzy Framework for the Electric Vehicle Selection

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ABSTRACT

Among many other selection criteria, multi-attribute decision-making is a tool used in various industries, including risk control, numerical examination, cybercrime investigation, networking, etc. Aczel-Alsina aggregation operators are suitable to mitigate the effects of inconsistent data. In this article, we present a few novel methodologies to account for T-spherical fuzzy set data using Aczel-Alsina aggregation techniques. The T-spherical fuzzy Aczel-Alsina weighted average operator and the T-spherical fuzzy Aczel-Alsina ordered weighted average are two examples of these novel, unique approaches. The T-spherical fuzzy Aczel-Alsina weighted geometric and T-spherical fuzzy Aczel-Alsina ordered weighted geometric operators are novel strategies that we also introduce. We look at a few unique situations and noteworthy properties to show the persistence and effectiveness of the techniques that are provided. A method for leveraging the T-spherical fuzzy information system to resolve the electric vehicle selection problem is also described. Due to the high cost of petrol and the current financial difficulties faced by middle-class households, this problem is chosen. An experimental case study is also built. To calculate the accuracy and authority of the present developments, we contrast the outcomes of formerly used methods with the aggregation operators that are presently available.

1. Introduction

Making decisions is difficult for many businesses, including persons in the fields of promotion, banking, town planning, and skills, among others. It is a prevalent delusion that all material about different accessibility applies to real-world situations. Zadeh [1] described the measured details of fuzzy sets (FSs), also recognizing its influence on many fields, embracing the theory of decision-making, association disciplines, manufacturing, computer knowledge, and numerous additional. The FS concept has some severe mistakes. One of these is the element that Zadeh only acquired in describing a purpose's membership degree (MD) in FSs. Atanassov [2] proposed a revolutionary idea

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known as intuitionistic FSs (IFSs). A non-membership degree (NMD) and MD, both of which have the same meaning as before, are distinguished by this useful tool. After inventing IFS, Yager [3,4] reduced the constraints on MD and NMD to generalize IFSs, which led to the creation of the Pythagorean FS (PyFS). Yager [5] introduced the idea of q-rung orthopair FS (q-ROFS). Additionally, the picture fuzzy set (PFS) was introduced by Cuong [6]. To do this, the PFS was enlarged to include three degrees of an element MD (NMD, neutral degree (ND), or abstention degree (AD). PFSs can accommodate different human opinions of acceptance, abstention, and rejection. Mahmood *et al.* [7] conducted research on the spherical FS (SFS) in 2019. Ullah *et al.* [8] expanded the SFS theory and introduced the idea of T-spherical fuzzy sets. This model expanded the SFS's range. Readers are directed to [9] to learn more about SFSs.

The concept of aggregation techniques is used by many research scientists, particularly those working in the areas of decision-making. Calvo *et al.* [10] developed theoretical concepts for weighted geometric operators and weighted averages. Xu [11] and Xu & Yager [12] research on the ideas of aggregation tools led to the development of a set of creative strategies for dealing with troubling and dubious data on human perspectives. Using linguistic IF information theory, Kumar & Chen [13] provided AOs to address the drawbacks of earlier methods and developed the multi-criteria group decision-making (MCGDM) method to address practical issues [14,15]. Within the constraints of IF circumstances, by merging two distinct approaches from the Schweizer-Sklar and Power combination tackles, Khan *et al.* [16] offered some new techniques. Rahim *et al.* [17] demonstrated several original techniques using PyFS data to solve a multi-attribute decision-making (MADM) problem. A Pythagorean triangle FSs to resolve an MADM problem was suggested in [18]. Khan *et al.* [19] used the ideas of hyper-soft ordered aggregation tools to provide a request for a green contractor organization based on the MADM method. Zhang *et al.* [20] looked into and suggested novel techniques for analyzing SF data based on the Dombi power Heronian mean. Riaz & Farid [21] constructed a novel AO structure based on SF conditions to manage complex and ambiguous input. PFSs were used as the foundation for other inventive techniques; i.e., the vendor management classification was applied by Naeem *et al.* [22]. Al-Quran *et al.* [23] discovered a usage for the travel and tourism industry and offered several inventive PFSs based on Hamy's mean tools. Aggregation algorithms were developed in the SFS environment to cope with ambiguity and impression data throughout the decision-making process.

Lately, a tilt of AOs for Aczel-Alsina aggregation techniques was put up. Additional helpful triangle criteria were examined and classified in [24]. Aczel-Alsina aggregation techniques are currently a prominent issue in research. Senapati *et al.* [25] created several aggregation methods using IFSs and the Aczel-Alsina theory. They also investigated the Aczel-Alsina triangle norm concepts in various fuzzy contexts [26,27] as well as presented some helpful methods [28,29]. Hussain *et al.* [30,31] produced various original methods using the fundamental techniques based on PyFS data from the Aczel-Alsina aggregation tools. In order to deal with issues in the real world, Khan *et al.* [32] built a test case and predicted particular actions based on the SFS aggregation operator (AO). To combat doubtful and unclear information, Khan *et al.* [33] developed acceptable T-SFS techniques based on SFSs. Riaz *et al.* [34] created some dependable and adaptable AOs of SFSs. Ullah *et al.* [35] identified several unique techniques and included some details about Aczel-Alsina aggregation-based T-SFSs. Under situations of the IF thinking, the notion of Heronian mean models was recognized by Hussain *et al.* [36]. In order to tackle experimental problems involving medical diagnosis, Ahmad *et al.* [37] established the notion of IF rough sets and put forth fresh ideas.

Using recently developed technology, Mehwish & Gul [38] amended the idea of new Hamacher AOs under the interval-valued complex T-spherical fuzzy environment. The Aczel-Alsina aggregation tools created by Sarfraz *et al.* [39] are used by some specific AOs. Yang *et al.* [40] used Einstein aggregation methods to develop the concept of SFSs and a specific set of AOs. Here are some noteworthy T-SFS-based traits and qualities. There are numerous simplifications of triangular norms, some of which include the Einstein aggregation model, the Frank aggregation tools, and the Dombi aggregation tools. Ullah *et al.* [41] used the Hamacher AOs to evaluate the effectiveness of search and rescue robots. In selecting solar cells, Zeng *et al.* [42] gave AOs on T-SFSs with an MADM solution. Zhang *et al.* [43] developed the consistency improvement for fuzzy preference in the application of two-sided matching decision making. Li *et al.* [44] gave the idea of decision making with incomplete hesitant fuzzy linguistic preference relations. Yang *et al.* [45] developed the new concept of decision making on Industry 4.0. Azem *et al.* [46] provided the concepts of metric-based resolvability of polycyclic aromatic hydrocarbons. Azem *et al.* [47] developed the M-polynomials based on nanostructures, while Azem *et al.* [48] focused on the localization of generalization hexagonal cellular networks. Munir *et al.* [49] constructed T-SF Einstein hybrid AOs and gave an illustration of their application in MADM. Making use of MADM, Mahnaz *et al.* [50] introduced T-SFs Frank AOs with their application in MADM. Mandal & Seikh [51] proposed a confidence level-driven Dombi AOs within the p,q-quasirung orthopair fuzzy environment.

The Hamacher aggregation techniques are among the several alternative triangular norm expansions. The extension of triangular norms indicated above is preferable for AOs. However, Aczel-Alsina aggregation processes are also superior. Aczel-Alsina aggregation methods provide accurate outcomes and ongoing approximation during the aggregation procedure. In any case, a lot of studies have been done on the concept of Aczel-Alsina aggregation procedures in many fuzzy fields. In this research, the T-SF information system is used to describe the core operational principles of the Aczel-Alsina aggregation apparatuses. We proposed a number of appropriate methods, including T-SF Aczel-Alsina weighted average (T-SFAWA), T-SF Aczel-Alsina ordered weighted average (T-SFAOWA), T-SF Aczel-Alsina weighted geometric (T-SFAWG), and T-SFs Aczel-Alsina ordered weighted geometric (T-SFAOWG) operators. There is also a brief review of some of the features of the present approaches. According to the previous analysis, AOs used in MADM are complicated by real-world phenomena. The information should be handled more readily in order to get the optimal alternative in MADM. Additionally, TSFSs operate in a more efficient ambiguity-handling environment than IFSs, PyFS, q-ROF, PFSs, and SFS. We have not yet discovered the use of the Aczel-Alsina AO within the T-SFS framework. These factors give us the motivation to formalize the idea of Aczel-Alsina AOs in the T-SFS layout and afterwards research their use in MADM. To assess the accuracy and dependability of the aggregation methods we have stated, we also give a detailed description of the comparative analyses, benefits, and drawbacks of our present research. Additionally, we provide an example using the electric vehicle selection problem.

The following is a list of this paper's contributions:

i. The T-SFAWA, T-SFAOWA, T-SFAWG, and T-SFAOWG operators are some of the new AOs we suggested for the T-SFS, and some relevant attributes are addressed.
ii. We created a brand-new T-SFS MADM approach.
iii. By addressing the electric vehicle selection problem, we evaluated the applicability of our suggested aggregation function-based MADM method.
iv. To demonstrate the suggested technique's dependability and efficacy, parameter analysis and comparison analysis are carried out with the existing approach.

In Section 1, we looked at the background of our present study projects and the aggregation measurement techniques that were available. Some T-SFS fundamentals and associated concepts are covered in Section 2. Using the Aczel-Alsina mathematical techniques is another method that Section 3 expands on the triangular norms. For instance, the T-SFAWA and T-SFAOWA operators are used in Section 4's pertinent AOs of T-SFSs that are based on Aczel-Alsina aggregation procedures. Section 5 also discussed the geometric aggregation theories of T-SFAWG and T-SFAOWG. The use of an MADM technique to select better electric vehicles based on our developed methodology is further examined in Section 6. The final section of our study contains requests, advantages of continuous work, and suggestions for the future.

2. Preliminaries

SFS, T-SFS, and the essential operations that apply to them are briefly described in this section. Based on accuracy and score values, the T-SF information comparison approach was examined.

Definition 1 [7]. A SFS is of the shape on set X= $\left\{\left(x,(s,i,d')\right): \forall\, x \in X\right\}$. Here $s, i,$ and d' are mappings from $X \to [0,1]$ denoting MD, AD, and ND, respectively, provided that $0 \leq sum\left(s^2(x), i^2(x), {d'}^2(x)\right) \leq 1$ and $r(x) = \sqrt{1 - sum\left(s^2(x), i^2(x), {d'}^2(x)\right)}$ is known as the RD of x in I. The triplet (s, i, d') is considered a spherical FV (SFV).

Definition 2 [8]. A T-SFS is of the shape on set $X = \left\{\left(x,(s,i,d')\right): \forall\, x \in X\right\}$. Here $s, i,$ and d' are mappings form $X \to [0,1]$ denoting MD, AD, and ND, respectively, provided that for some $\mathbb{T} \in z^+$ $0 \leq sum\left(s^{\mathbb{T}}(x), i^{\mathbb{T}}(x), {d'}^{\mathbb{T}}(x)\right) \leq 1$ and $r(x) = \sqrt{1 - sum\left(s^{\mathbb{T}}(x), i^{\mathbb{T}}(x), {d'}^{\mathbb{T}}(x)\right)}$ is known as the RD of x in I. The triplet (s, i, d') is considered as a T-spherical FV (T-SFV).

Remark 1.

i. When $\mathbb{T} = 2$, T-SFS degenerates to SFS.
ii. When $\mathbb{T} = 1$, T-SFS transforms to PFS.
iii. If $\mathbb{T} = 2$ and $i = 0$, T-SFS degenerates to PyFS.
iv. If $\mathbb{T} = 1$ and $i = 0$, T-SFS degenerates to IFS.
v. When $\mathbb{T} = 1$ and $i = d = 0$, T-SFS degenerates FS.

Definition 3 [12]. Let $\beta_1 = (s_1, i_1, {d'}_1)$ be a T-SFV. Then:

$$\boldsymbol{Sco(\beta_1) = (s_1 - i_1 - {d'}_1)}, \tag{1}$$

is the score value of β_1.

Definition 4 [12]. Let $\beta_1 = (s_1, i_1, {d'}_1)$ be an IFV. Then:

$$\boldsymbol{Acc(\beta_1) = (s_1 + i_1 + {d'}_1)}, \tag{2}$$

is the degrees of accuracy of β_1. Besides:

i. If $Sco(\beta_1) < Sco(\beta_2)$, then $\beta_1$has less preference than β_2.
ii. If $Sco(\beta_1) = Sco(\beta_2)$, then $\beta_1$and β_2 are the same.
iii. If $Acc\ (\beta_1) < Acc\ (\beta_2)$, then $\beta_1$has less preference than β_2.
iv. If $Acc\ (\beta_1) = Acc\ (\beta_2)$, then $\beta_1$and β_2 are the same.

Definition 5 [52]**.** The Aczel-Alsina t-norms $\left(T_A^{\varsigma}\right)_{\varsigma\epsilon[0,\infty]}$ are ascertained by

$$\left(\boldsymbol{T_A^{\varsigma}}\right)_{(\boldsymbol{\ell},\boldsymbol{v})}=\begin{cases}\boldsymbol{T_D(\ell,v)}\ \textbf{if}\ \boldsymbol{\varsigma}=\mathbf{0}\\ \boldsymbol{min(\ell,v)}\ \textbf{if}\ \boldsymbol{\varsigma}=\infty\\ \boldsymbol{e}^{-((-\boldsymbol{ln\ell})^{\varsigma}+(-\boldsymbol{lnv})^{\varsigma})^{\frac{1}{\varsigma}}}\ \textbf{otherwise}\end{cases}. \tag{3}$$

The Aczel-Alsina t-norms $\left(S_A^{\varsigma}\right)_{\varsigma\epsilon[0,\infty]}$ are ascertained by:

$$\left(\boldsymbol{S_A^{\varsigma}}\right)_{(\boldsymbol{\ell},\boldsymbol{v})}=\begin{cases}\boldsymbol{S_D(\ell,v)}\ \boldsymbol{if}\ \boldsymbol{\varsigma}=\mathbf{0}\\ \boldsymbol{max(\ell,v)}\ \textbf{if}\ \boldsymbol{\varsigma}=\infty\\ \mathbf{1}-\boldsymbol{e}^{-\left(\left(-\boldsymbol{ln}(\mathbf{1}-\boldsymbol{\ell})\right)^{\varsigma}+\left(-\boldsymbol{ln}(\mathbf{1}-\boldsymbol{v})\right)^{\varsigma}\right)^{\frac{1}{\varsigma}}}\ \textbf{otherwise}\end{cases}, \tag{4}$$

where, the limiting values are $T_A^{\infty}=min$, $T_A^{0}=T_D$, $T_A^{1}=T_p$, $S_A^{\infty}=max$, $S_A^{0}=S_D$, and $S_A^{1}=S_p$.

Definition 6 [12]. Let $\beta=(s,i,d')$, $\beta_1=(s_1,i_1,d'_1)$, and $\beta_2=(s_2,i_2,d'_2)$ be three T-SFVs and $\Phi\geq 0$. Then, we have:

$$\boldsymbol{\beta_1}\oplus\boldsymbol{\beta_2}=\left(\sqrt{\boldsymbol{s}^{\mathbb{T}}+\boldsymbol{s}^{\mathbb{T}}-\boldsymbol{s}^{\mathbb{T}}\boldsymbol{s}^{\mathbb{T}}},\boldsymbol{i}^{\mathbb{T}}.\boldsymbol{i}^{\mathbb{T}},\boldsymbol{d'}^{\mathbb{T}}\boldsymbol{d'}^{\mathbb{T}}\right), \tag{5}$$

$$\boldsymbol{\beta_1}\otimes\boldsymbol{\beta_2}=\left(\boldsymbol{s}^{\mathbb{T}}\boldsymbol{d'}^{\mathbb{T}},\sqrt{\boldsymbol{i}^{\mathbb{T}}+\boldsymbol{i}^{\mathbb{T}}-\boldsymbol{i}^{\mathbb{T}}.\boldsymbol{i}^{\mathbb{T}}},\sqrt{\boldsymbol{d'}^{\mathbb{T}}+\boldsymbol{d'}^{\mathbb{T}}-\boldsymbol{d'}^{\mathbb{T}}\boldsymbol{d'}^{\mathbb{T}}}\right), \tag{6}$$

$$\boldsymbol{\Phi\beta}=\left(\sqrt{\mathbf{1}-(\mathbf{1}-\boldsymbol{s}^{\mathbb{T}})^{\varsigma}},\ \boldsymbol{i}^{\varsigma},\boldsymbol{d'}^{\varsigma}\right), \tag{7}$$

$$\boldsymbol{\beta}^{\boldsymbol{\Phi}}=\left(\boldsymbol{s}^{\varsigma},\sqrt{\mathbf{1}-(\mathbf{1}-\boldsymbol{i}^{\mathbb{T}})^{\varsigma}},\sqrt{\mathbf{1}-\left(\mathbf{1}-\boldsymbol{d'}^{\mathbb{T}}\right)^{\varsigma}}\right). \tag{8}$$

3. Aczel-Alsina Operations for T-SFVS

Based on T-SF data, we gave a thorough explanation of the fundamental operating principles of the Aczel-Alsina aggregation models.

Definition 7 [51]. Let $\beta=(s,i,d')$, $\beta_1=(s_1,i_1,d'_1)$, and $\beta_2=(s_2,i_2,d'_2)$ be three T-SFVs, $\varsigma\geq 1$, and $\Phi\geq 0$. Then, the Aczel-Alsina t-norm and t-conorm operations of T-SFVs are distinct as:

$$\boldsymbol{\beta_1}\oplus\boldsymbol{\beta_2}=\begin{pmatrix}\sqrt[\mathbb{T}]{\mathbf{1}-\boldsymbol{e}^{-\left(\left(-\boldsymbol{log}\left(\mathbf{1}-\boldsymbol{s}_{\mathbf{1}}^{\mathbb{T}}\right)\right)^{\varsigma}+\left(-\boldsymbol{log}\left(\mathbf{1}-\boldsymbol{s}_{\mathbf{2}}^{\mathbb{T}}\right)\right)^{\varsigma}\right)^{\frac{1}{\varsigma}}}},\\ \boldsymbol{e}^{-\left(\left(-\boldsymbol{log}(\boldsymbol{i}_{\mathbf{1}})\right)^{\varsigma}+\left(-\boldsymbol{log}(\boldsymbol{i}_{\mathbf{2}})\right)^{\varsigma}\right)^{\frac{1}{\varsigma}}},\boldsymbol{e}^{-\left(\left(-\boldsymbol{log}(\boldsymbol{d'}_{\mathbf{1}})\right)^{\varsigma}+\left(-\boldsymbol{log}(\boldsymbol{d'}_{\mathbf{2}})\right)^{\varsigma}\right)^{\frac{1}{\varsigma}}}\end{pmatrix}, \tag{9}$$

$$\boldsymbol{\beta_1}\otimes\boldsymbol{\beta_2}=\begin{pmatrix}\boldsymbol{e}^{-\left(\left(-\boldsymbol{log}(\boldsymbol{s}_{\mathbf{1}})\right)^{\varsigma}+\left(-\boldsymbol{log}(\boldsymbol{s}_{\mathbf{2}})\right)^{\varsigma}\right)^{\frac{1}{\varsigma}}},\\ \sqrt[\mathbb{T}]{\mathbf{1}-\boldsymbol{e}^{-\left(\left(-\boldsymbol{log}\left(\mathbf{1}-\boldsymbol{i}_{\mathbf{1}}^{\mathbb{T}}\right)\right)^{\varsigma}+\left(-\boldsymbol{log}\left(\mathbf{1}-\boldsymbol{i}_{\mathbf{2}}^{\mathbb{T}}\right)\right)^{\varsigma}\right)^{\frac{1}{\varsigma}}}},\sqrt[\mathbb{T}]{\mathbf{1}-\boldsymbol{e}^{-\left(\left(-\boldsymbol{log}\left(\mathbf{1}-\boldsymbol{d'}_{\mathbf{1}}^{\mathbb{T}}\right)\right)^{\varsigma}+\left(-\boldsymbol{log}\left(\mathbf{1}-\boldsymbol{d'}_{\mathbf{2}}^{\mathbb{T}}\right)\right)^{\varsigma}\right)^{\frac{1}{\varsigma}}}}\end{pmatrix}, \tag{10}$$

$$\boldsymbol{\Phi\beta}=\left(\sqrt[\mathbb{T}]{\mathbf{1}-\boldsymbol{e}^{-\left(\boldsymbol{\Phi}\left(-\boldsymbol{log}(\mathbf{1}-\boldsymbol{s}^{\mathbb{T}})\right)^{\varsigma}\right)^{\frac{1}{\varsigma}}}},\ \boldsymbol{e}^{-\left(\boldsymbol{\Phi}\left(-\boldsymbol{log}(\boldsymbol{i})\right)^{\varsigma}\right)^{\frac{1}{\varsigma}}},\boldsymbol{e}^{-\left(\boldsymbol{\Phi}\left(-\boldsymbol{log}(\boldsymbol{d}')\right)^{\varsigma}\right)^{1/\varsigma}}\right), \tag{11}$$

$$\boldsymbol{\beta}^{\boldsymbol{\Phi}}=\left(\boldsymbol{e}^{-\left(\boldsymbol{\Phi}\left(-\boldsymbol{log}(\boldsymbol{s})\right)^{\varsigma}\right)^{1/\varsigma}},\sqrt[\mathbb{T}]{\mathbf{1}-\boldsymbol{e}^{-\left(\boldsymbol{\Phi}\left(-\boldsymbol{log}(\mathbf{1}-\boldsymbol{i}^{\mathbb{T}})\right)^{\varsigma}\right)^{\frac{1}{\varsigma}}}},\sqrt[\mathbb{T}]{\mathbf{1}-\boldsymbol{e}^{-\left(\boldsymbol{\Phi}\left(-\boldsymbol{log}\left(\mathbf{1}-\boldsymbol{d}'^{\mathbb{T}}\right)\right)^{\varsigma}\right)^{1/\varsigma}}}\right). \tag{12}$$

4. Aczel-Alisna Averaging Aggregations Operations for T-SFVs

In this sector, we mentioned a number of innovative T-SF strategies. The related weight vector of this article β_j is represented by $K_j = K_1 \oplus K_2 \oplus \ldots \oplus K_\vartheta$, $j = 1, 2,3, \ldots, \vartheta$, such that $K \in [0, 1]$ and $\sum_{\tau=1}^{\vartheta} K_j$.

Definition 8. Let $\beta_j = \left(s_j, i_j, d'_j\right)$ $j = 1, 2, \ldots, \vartheta$ be the set of T-SFVs. Then, the T-SFAAWA operator is particularized as:

$$\boldsymbol{T}-\boldsymbol{SFAAWA}(\boldsymbol{\beta}_1,\boldsymbol{\beta}_2,\ldots,\boldsymbol{\beta}_{\vartheta})=\boldsymbol{K}_1\boldsymbol{\beta}_1\oplus\boldsymbol{K}_2\boldsymbol{\beta}_2\oplus\ldots\oplus\boldsymbol{K}_{\vartheta}\boldsymbol{\beta}_{\vartheta}=\oplus_{j=1}^{\vartheta}\boldsymbol{K}_j\boldsymbol{\beta}_j. \tag{13}$$

We obtain the subsequent theorem through Aczel-Alsina operations on T-SFVs.

Theorem 1. Let $\beta_j = \left(s_j, i_j, d'_j\right)$ be the set of T-SFVs. T-SFV obtained from the sum of the values of the T-SFAAWA operator is then:

$$\boldsymbol{T}-\boldsymbol{SFAAWA}(\boldsymbol{\beta}_1,\boldsymbol{\beta}_2,\ldots\boldsymbol{\beta}_{\vartheta})=\begin{pmatrix}\sqrt[\mathbb{T}]{\mathbf{1}-\boldsymbol{e}^{-\left(\sum_{j=1}^{\vartheta}\left(\boldsymbol{K}_j\left(-\boldsymbol{log}\left(\mathbf{1}-\boldsymbol{s}_j^{\mathbb{T}}\right)\right)^{\varsigma}\right)\right)^{\frac{1}{\varsigma}}}},\\ \boldsymbol{e}^{-\left(\sum_{j=1}^{\vartheta}\left(\boldsymbol{K}_j\left(-\boldsymbol{log}(\boldsymbol{i}_j)\right)^{\varsigma}\right)\right)^{\frac{1}{\varsigma}}},\boldsymbol{e}^{-\left(\boldsymbol{K}_j\left(\sum_{j=1}^{\vartheta}\left(-\boldsymbol{log}(\boldsymbol{d}'_j)\right)^{\varsigma}\right)\right)^{\frac{1}{\varsigma}}}\end{pmatrix}. \tag{14}$$

Proof of Theorem 1 is provided in Appendix 1.

Example 1. Consider four T-SFVs are $\beta_1 = (0.67, 0.56, 0.55)$, $\beta_2 = (0.92, 0.56, 0.41)$, $\beta_3 = (0.93, 0.33, 0.45)$, and $\beta_4 = (0.98, 0.49, 0.33)$. Let $K = (0.20, 0.25, 0.3, 0.25)$ be the weight vector of the given T-SFVs, with $\mathbb{T} = 4$ and $\varsigma = 3$. Next, the T-SFAAWA operators' answer to the given aggregated values is given as:

$$\boldsymbol{T}-\boldsymbol{SFAAWA}=\begin{pmatrix}\sqrt[4]{\mathbf{1}-\boldsymbol{e}^{-\left(\left((0.20)\left(-\boldsymbol{log}(1-0.67^4)\right)^3\right)+\left((0.25)\left(-\boldsymbol{log}(1-0.92^4)\right)^3\right)+\left((0.30)\left(-\boldsymbol{log}(1-0.93^4)\right)^3\right)+\left((0.25)\left(-\boldsymbol{log}(1-0.98^4)\right)^3\right)\right)^{\frac{1}{3}}}},\\ \boldsymbol{e}^{-\left(\left((0.20)\left(-\boldsymbol{log}(0.56)\right)^3\right)+\left((0.25)\left(-\boldsymbol{log}(0.56)\right)^3\right)+\left((0.20)\left(-\boldsymbol{log}(0.33)\right)^3\right)+\left((0.20)\left(-\boldsymbol{log}(0.49)\right)^3\right)\right)^{\frac{1}{3}}},\\ \boldsymbol{e}^{-\left(\left((0.20)\left(-\boldsymbol{log}(0.55)\right)^3\right)+\left((0.25)\left(-\boldsymbol{log}(0.41)\right)^3\right)+\left((0.20)\left(-\boldsymbol{log}(0.45)\right)^3\right)+\left((0.20)\left(-\boldsymbol{log}(0.33)\right)^3\right)\right)^{\frac{1}{3}}}\end{pmatrix}$$

$= (0.7312, 0.6951, 0.6783)$.

Theorem 2 (idempotency). If all $\beta_j = \left(s_j, i_j, d'_j\right)$ are the set of all equal T-SFVs that are $\beta_j = \beta$ for all j. Then:

$$\boldsymbol{T}-\boldsymbol{SFAAWA}(\boldsymbol{\beta}_1,\boldsymbol{\beta}_2,\ldots,\boldsymbol{\beta}_{\vartheta})=\boldsymbol{\beta}. \tag{15}$$

Proof of Theorem 2 is provided in Appendix 2.

Theorem 3 (monotonicity). Let $\beta_j = \left(s_j, i_j, d'_j\right)$ be a set of T-SFVs. If $\beta^- = min(\beta_1, \beta_2, \dots, \beta_\vartheta)$ and $\beta^+ = max(\beta_1, \beta_2, \dots, \beta_\vartheta)$, then:

$$\boldsymbol{\beta^- \leq T - SFAAWA(\beta_1, \beta_2, \dots, \beta_\vartheta) \leq \beta^+}. \tag{16}$$

Proof of Theorem 3 is provided in Appendix 3.

Theorem 4 (boundedness). Let β_j and β'_j be two sets of T-SFVs. If $\beta_j \leq \beta'_j$ for all j, then:

$$\boldsymbol{T - SFAAWA(\beta_1, \beta_2, \dots, \beta_\vartheta) \leq T - SFAAWA(\beta'_1, \beta'_2, \dots, \beta'_\vartheta)}. \tag{17}$$

Proof of Theorem 4. Straightforward.

Definition 9. Let $\beta_j = \left(s_j, i_j, d'_j\right)$ $j = 1, 2, \dots, \vartheta$ be the collection of T-SFVs. The T-SFAAOWA operator is then described in more detail as:

$$\boldsymbol{T - SFAAOWA(\beta_1, \beta_2, \dots \beta_\vartheta) = K_1 \beta_{p(1)} \oplus K_2 \beta_{p(2)} \oplus \dots \oplus K_\vartheta \beta_{p(\vartheta)} = \oplus_{j=1}^{\vartheta} K_j \beta_{p(j)}}, \tag{18}$$

where $\left(p(1), p(2), \dots, p(j)\right)$ are a collection of T-SFNs.

Theorem 5. Let $\beta_j = \left(s_j, i_j, d'_j\right)$ be the set of T-SFVs. Then, the united value of the T-SFAAOWA operator stills a T-SFV given by:

$$\boldsymbol{T - SFAAOWA(\beta_1, \beta_2, \dots \beta_\vartheta)} = \begin{pmatrix} \sqrt[\mathbb{T}]{\boldsymbol{1 - e^{-\left(\sum_{j=1}^{\vartheta}\left(K_j\left(-log\left(1 - s_{p(j)}^{\mathbb{T}}\right)\right)^{\varsigma}\right)\right)^{\frac{1}{\varsigma}}}}}, \\ \boldsymbol{e^{-\left(\sum_{j=1}^{\vartheta}\left(K_j\left(-log\left(i_{p(j)}\right)\right)^{\varsigma}\right)\right)^{\frac{1}{\varsigma}}}, e^{-\left(K_j\left(\sum_{j=1}^{\vartheta}\left(-log\left(d'p(j)\right)\right)^{\varsigma}\right)\right)^{\frac{1}{\varsigma}}}} \end{pmatrix}, \tag{19}$$

where $\left(p(1), p(2), \dots, p(j)\right)$ be the set of permutations of $j = 1, 2, \dots, \vartheta$ and $\beta_{p(\vartheta-1)} \geq \beta_{p(\vartheta)}, \forall, j = 1, 2, \dots, \vartheta$.

Theorem 6 (idempotency). Consider $\beta_j = \left(s_j, i_j, d'_j\right)$ be the family of all the same T-SFVs that is $\beta_j = \beta$ for all j. Then:

$$\boldsymbol{T - SFAAOWA(\beta_1, \beta_2, \dots, \beta_\vartheta) = \beta}. \tag{20}$$

Theorem 7 (monotonicity). Consider $\beta_j = \left(s_j, i_j, d'_j\right)$ be family of all same T-SFVs. If $\beta^- = min(\beta_1, \beta_2, \dots, \beta_\vartheta)$ and $\beta^+ = max(\beta_1, \beta_2, \dots, \beta_\vartheta)$, then:

$$\boldsymbol{\beta^- \leq T - SFAAOWA(\beta_1, \beta_2, \dots, \beta_\vartheta) \leq \beta^+}. \tag{21}$$

Theorem 8 (boundedness). Let β_j and β'_j be a family of all the same T-SFVs. If $\beta_j \leq \beta'_j$ for all j, then:

$$\boldsymbol{T - SFAAOWA(\beta_1, \beta_2, \dots, \beta_\vartheta) \leq T - SFAAOWA(\beta'_1, \beta'_2, \dots, \beta'_\vartheta)}. \tag{22}$$

5. Aczel-Alsina Geometric Aggregations Operations for T-SFVs

The basic operations of the Aczel-Alsina aggregation tools were used in this part to construct a few geometric aggregation tools for T-SFVs.

Definition 10. Let $\beta_j = (s_j, i_j, d'_j)$ $j = 1, 2, \dots, \vartheta$ be the set of T-SFVs. Then, the T-SFAAWA operator is particularized as:

$$\boldsymbol{T - SFAAWG(\beta_1, \beta_2, \dots \beta_\vartheta) = K_1^{\beta_1} \otimes K_2^{\beta_2} \otimes \dots \otimes K_\vartheta^{\beta_\vartheta} = \otimes_{j=1}^{\vartheta} K_j^{\beta_j}}. \tag{23}$$

We obtain the subsequent theorem by means of Aczel-Alsina operations on T-SFVs.

Theorem 9. Let $\beta_j = (s_j, i_j, d'_j)$ be the set of T-SFVs. The T-SFV provided by the combined value of the T-SFAAWG operator is then:

$$\boldsymbol{T - SFAAWG(\beta_1, \beta_2, \dots \beta_\vartheta)} = \begin{pmatrix} \boldsymbol{1 - e^{-\left(\sum_{j=1}^{\vartheta}\left(K_j\left(-log(1-s_j)\right)^{\varsigma}\right)\right)^{\frac{1}{\varsigma}}}}, \\ \sqrt[\mathbb{T}]{\boldsymbol{e^{-\left(\sum_{j=1}^{\vartheta}\left(K_j\left(-log\left(i_j^{\mathbb{T}}\right)\right)^{\varsigma}\right)\right)^{\frac{1}{\varsigma}}}}}, \sqrt[\mathbb{T}]{\boldsymbol{e^{-\left(K_j\left(\sum_{j=1}^{\vartheta}\left(-log\left(d'^{\mathbb{T}}_j\right)\right)^{\varsigma}\right)\right)^{\frac{1}{\varsigma}}}}} \end{pmatrix}. \tag{24}$$

Example 2. Consider four T-SFVs are $\beta_1 = (0.67, 0.56, 0.55)$, $\beta_2 = (0.92, 0.56, 0.41)$, $\beta_3 = (0.93, 0.33, 0.45)$, and $\beta_4 = (0.98, 0.49, 0.33)$. Let $K = (0.20, 0.25, 0.3, 0.25)$ be the specified T-SFV weight vector, with $\mathbb{T} = 4$ and $\varsigma = 3$. Next, the T-SFAAWA operators' responses to the given aggregated values are given as:

$$\boldsymbol{T - SFAAWA} = \begin{pmatrix} \boldsymbol{e^{-\left(\left((0.20)(-log(0.67))^3\right)+\left((0.25)(-log(0.92))^3\right)+\left((0.30)(-log(0.93))^3\right)+\left((0.25)(-log(0.98))^3\right)\right)^{\frac{1}{3}}}}, \\ \sqrt[4]{\boldsymbol{1 - e^{-\left(\left((0.20)\left(-log(1-0.56^4)\right)^3\right)+\left((0.25)\left(-log(1-0.56^4)\right)^3\right)+\left((0.20)\left(-log(1-0.33^4)\right)^3\right)+\left((0.20)\left(-log(1-0.49^4)\right)^3\right)\right)^{\frac{1}{3}}}}} \\ \sqrt[4]{\boldsymbol{1 - e^{-\left(\left((0.20)\left(-log(1-0.55^4)\right)^3\right)+\left((0.25)\left(-log(1-0.41^4)\right)^3\right)+\left((0.20)\left(-log(1-0.45^4)\right)^3\right)+\left((0.20)\left(-log(1-0.33^4)\right)^3\right)\right)^{\frac{1}{3}}}}} \end{pmatrix}$$

$= (0.9026, 0.1870, 0.1590)$.

Theorem 10 (idempotency). If all $\beta_j = (s_j, i_j, d'_j)$ are the set of all equal T-SFVs that are $\beta_j = \beta$ for all j. Then:

$$\boldsymbol{T - SFAAWG(\beta_1, \beta_2, \dots, \beta_\vartheta) = \beta}. \tag{25}$$

Theorem 11 (monotonicity). Let $\beta_j = (s_j, i_j, d'_j)$ be a set of T-SFVs. If $\beta^- = min(\beta_1, \beta_2, \dots, \beta_\vartheta)$ and $\beta^+ = max(\beta_1, \beta_2, \dots, \beta_\vartheta)$, then:

$$\boldsymbol{\beta^- \leq T - SFAAWG(\beta_1, \beta_2, \dots, \beta_\vartheta) \leq \beta^+}. \tag{26}$$

Theorem 12 (boundedness). Let β_j and β'_j be two sets of T-SFVs. If $\beta_j \leq \beta'_j$ for all j, then:

$$\boldsymbol{T - SFAAWG(\beta_1, \beta_2, \dots, \beta_\vartheta) \leq T - SFAAWG(\beta'_1, \beta'_2, \dots, \beta'_\vartheta)}. \tag{27}$$

Theorem 13. Let $\beta_j = (s_j, i_j, d'_j)$ be the collection of T-SFVs. The T-SFV provided by the combined value of the T-SFAAOWG operator is then:

$$\boldsymbol{T-SFAAOWG(\beta_1,\beta_2,\dots\beta_\vartheta)} = \left(\begin{array}{c} \mathbf{1}-e^{-\left(\sum_{j=1}^{\vartheta}\left(K_j\left(-log\left(1-s_{p(j)}\right)\right)^{\varsigma}\right)\right)^{\frac{1}{\varsigma}}}, \\ \sqrt[\mathbb{T}]{e^{-\left(\sum_{j=1}^{\vartheta}\left(K_j\left(-log\left(i_{p(j)}^{\mathbb{T}}\right)\right)^{\varsigma}\right)\right)^{\frac{1}{\varsigma}}}}, \sqrt[\mathbb{T}]{e^{-\left(K_j\left(\sum_{j=1}^{\vartheta}\left(-log\left(d'^{\mathbb{T}}_{p(j)}\right)\right)^{\varsigma}\right)\right)^{\frac{1}{\varsigma}}}} \end{array}\right), \tag{28}$$

where $\left(p(1),p(2),\dots,p(j)\right)$ be the set of permutations of $j=1,2,\dots,\vartheta$ and $\beta_{p(\vartheta-1)}\geq\beta_{p(\vartheta)},\forall,j=1,2,\dots,\vartheta$.

Theorem 14 (idempotency). Consider $\beta_j=\left(s_j,i_j,d'_j\right)$ be the family of all the same T-SFVs that is $\beta_j=\beta$ for all j. Then:

$$\boldsymbol{T-SFAAOWG(\beta_1,\beta_2,\dots,\beta_\vartheta)=\beta}. \tag{29}$$

Theorem 15 (monotonicity). Consider $\beta_j=\left(s_j,i_j,d'_j\right)$ be a family of all the same T-SFVs. If $\beta^-=min(\beta_1,\beta_2,\dots,\beta_\vartheta)$ and $\beta^+=max(\beta_1,\beta_2,\dots,\beta_\vartheta)$, then:

$$\boldsymbol{\beta^-\leq T-SFAAOWG(\beta_1,\beta_2,\dots,\beta_\vartheta)\leq\beta^+}. \tag{30}$$

Theorem 16 (boundedness). Let β_j and β'_j be a family of all the same T-SFVs. If $\beta_j\leq\beta'_j$ for all j, then:

$$\boldsymbol{T-SFAAOWG(\beta_1,\beta_2,\dots,\beta_\vartheta)\leq T-SFAAOWG(\beta'_1,\beta'_2,\dots,\beta'_\vartheta)}. \tag{31}$$

6. Approach of the Electric Vehicle Selection using T-SF Data

We now evaluate the MADM problem employing the T-SFAAWA and T-SFAAWG operators. Our established methods are used to participate in the material gathering, and afterwards, laborious study and data processing, we arrive at a single value for a potent decision-making persistence. These methods make material evaluation simpler and more accurate.

Consider a set of alternatives that includes various alternative kinds, such as $\left\{A_1,A_2,\dots,A_{\dot{\rho}}\right\}$and a set of attributes that includes distinct features, such as $\{J_1,J_2,\dots,J_4\}$. A decision-maker assesses data by the relative weights that experts have given to each attribute. Assume that a collection of traits to the degree represented by the $đ_s=\left(đ_1,đ_2,\dots,đ_ȝ\right)$ such that $đ_s>0$ and $\sum_{s=1}^{ȝ}đ_i=1$. Based on the experts' grading of characteristics, decision-makers evaluate the information of T-SFVs with $\beta_{ps}=\left(s_{ps},i_{ps},d'_{ps}\right)$, $p=1,2,\dots,\dot{\rho}$ and $=1,2,\dots,ȝ$. Each T-SFV satisfies a condition $0\leq s_{ps}+i_{ps}+d'_{ps}\leq 1$. Based on various crucial informational elements, such as options and qualities, the decision maker created T-SF information in the decision matrix. Experts analyzed the information gathering and computed the results of individuals based on various attributes in a single term by using specified approaches with assigning degrees of characteristics. Using the subsequent steps of an algorithm under our now-provided approaches, we investigated an algorithm to assess the electric vehicle selection problem.

Step 1: The decision maker provides information for each choice in the form of T-SFVs so that it can be evaluated according to certain standards. The following decision matrix is created using all the data gathered:

$$\boldsymbol{\varphi} = \left(\boldsymbol{\beta}_{\boldsymbol{ps}}\right)_{\dot{\rho}\times \mathbf{3}} = \begin{bmatrix} \boldsymbol{\beta}_{\mathbf{11}} & \boldsymbol{\beta}_{\mathbf{12}} & \cdots & \boldsymbol{\beta}_{\mathbf{13}} \\ \boldsymbol{\beta}_{\mathbf{21}} & \boldsymbol{\beta}_{\mathbf{22}} & \cdots & \boldsymbol{\beta}_{\mathbf{23}} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{\beta}_{\dot{\rho}} & \boldsymbol{\beta}_{\dot{\rho}} & \cdots & \boldsymbol{\beta}_{\dot{\rho}} \end{bmatrix}, \tag{32}$$

where $\left(\beta_{ps}\right)_{\dot{\rho}\times 3} = \left(s_{ps}, i_{ps}, d'_{ps}\right)_{\dot{\rho}\times 3}, p = 1, 2, \ldots, \dot{\rho}$ and $s = 1, 2, \ldots, 3$.

Step 2: Utilizing the T-SFAAWA and T-SFAAWG operators' newly developed methods, integrate each attribute related to each possibility depending on specific criteria:

$$\boldsymbol{T-SFAAWA}(\boldsymbol{\beta}_{\mathbf{1}}, \boldsymbol{\beta}_{\mathbf{2}}, \ldots \boldsymbol{\beta}_{\boldsymbol{\vartheta}}) = \left(\begin{array}{c} \sqrt[\mathbb{T}]{\mathbf{1} - \boldsymbol{e}^{-\left(\sum_{j=1}^{\vartheta}\left(K_j\left(-\boldsymbol{log}\left(\mathbf{1}-s_j^{\mathbb{T}}\right)\right)^{\varsigma}\right)\right)^{\frac{1}{\varsigma}}}}, \\ \boldsymbol{e}^{-\left(\sum_{j=1}^{\vartheta}\left(K_j\left(-\boldsymbol{log}(i_j)\right)^{\varsigma}\right)\right)^{\frac{1}{\varsigma}}}, \boldsymbol{e}^{-\left(K_j\left(\sum_{j=1}^{\vartheta}\left(-\boldsymbol{log}(d'_j)\right)^{\varsigma}\right)\right)^{\frac{1}{\varsigma}}} \end{array} \right), \tag{33}$$

$$\boldsymbol{T-SFAAWG}(\boldsymbol{\beta}_{\mathbf{1}}, \boldsymbol{\beta}_{\mathbf{2}}, \ldots \boldsymbol{\beta}_{\boldsymbol{\vartheta}}) = \left(\begin{array}{c} \mathbf{1} - \boldsymbol{e}^{-\left(\sum_{j=1}^{\vartheta}\left(K_j\left(-\boldsymbol{log}(\mathbf{1}-s_j)\right)^{\varsigma}\right)\right)^{\frac{1}{\varsigma}}}, \\ \sqrt[\mathbb{T}]{\boldsymbol{e}^{-\left(\sum_{j=1}^{\vartheta}\left(K_j\left(-\boldsymbol{log}\left(i_j^{\mathbb{T}}\right)\right)^{\varsigma}\right)\right)^{\frac{1}{\varsigma}}}}, \sqrt[\mathbb{T}]{\boldsymbol{e}^{-\left(K_j\left(\sum_{j=1}^{\vartheta}\left(-\boldsymbol{log}\left(d'^{\mathbb{T}}_j\right)\right)^{\varsigma}\right)\right)^{\frac{1}{\varsigma}}}} \end{array} \right). \tag{34}$$

Step 3: Based on the effects of the T-SFAAWA and T-SFAAWG operators, compute the score value.

Step 4: Rearrange all score values in descending order to evaluate a better alternative.

6.1. Results

Modern life depends on transportation, yet combustion engines are getting older. Since fully electric vehicles are so much more ecologically friendly, they will soon replace petrol and diesel-powered automobiles. Comparable petrol or diesel automobiles have much higher operational costs than electric vehicles. Instead of using fossil fuels like petrol or diesel, electric vehicles use electricity to recharge their batteries. Electric vehicles are more cost-effective than others because charging one costs less than buying petrol or fuel to control it for our mobility needs. Electric automobiles can be used in a more environmentally beneficial way if they are driven by renewable energy. If accusing is done with the aid of renewable energy sources connected at home, such as solar, the cost of electricity can be further decreased.

Think about the five distinct sorts of electric vehicles that are available $(A_1, A_2, \ldots, A_5)$. The attributes are used by the decision-maker to evaluate these electric vehicles, including $J_{1,}$ which denotes the battery storage capacity, J_2 which denotes the low maintenance costs, J_3 which is a dependable and user-friendly system, and J_4 which indicates the long warranty and low noise pollution rate. The decision maker's characteristic degrees are used to evaluate each of the features that have been discussed in the weight vector $(0.20, 0.25, 0.30, 0.25)$. T-SFVs are used to represent data on electric vehicles in the decision matrix that follows.

Step 1: Obtained T-SF data regarding electric motor vehicles is shown in Table 1.

Table 1
Information based on T-SFVs

	J_1	J_2	J_3	J_4
A_1	$(0.92,0.42,0.33)$	$(0.91,0.45,0.51)$	$(0.96,0.43,0.13)$	$(0.92,0.23,0.45)$
A_2	$(0.88,0.23,0.61)$	$(0.81,0.45,0.45)$	$(0.83,0.56,0.34)$	$(0.94,0.53,0.44)$
A_3	$(0.76,0.66,0.55)$	$(0.85,0.62,0.17)$	$(0.87,0.57,0.27)$	$(0.88,0.29,0.76)$
A_4	$(0.66,0.56,0.51)$	$(0.77,0.46,0.45)$	$(0.94,0.42,0.55)$	$(0.87,0.29,0.67)$
A_5	$(0.82,0.56,0.32)$	$(0.78,0.44,0.55)$	$(0.95,0.38,0.29)$	$(0.89,0.47,0.33)$

Step 2: Table 2 illustrates the results produced by the T-SFAAWA and T-SFAAWG operators.

Table 2
Results of the T-SFAAWA and T-SFAAWG operators

	T-SFAAWA	T-SFAAWG
A_1	$(0.6906,0.6278,0.5336)$	$(0.9663,0.120,0.1423)$
A_2	$(0.6047,0.6591,0.6843)$	$(0.9296,0.1835,0.1943)$
A_3	$(0.5385,0.6977,0.5728)$	$(0.9220,0.2502,0.3247)$
A_4	$(0.6096,0.6633,0.7556)$	$(0.8896,0.1649,0.2490)$
A_5	$(0.6380,0.6996,0.6257)$	$(0.9248,0.1664,0.1611)$

Step 3: All calculated consequences for the score values are shown in Table 3.

Table 3
Score values of T-SFAAWA and T-SFAAWG

	T-SFAAWA	T-SFAAWG
A_1	-0.4708	0.7040
A_2	-0.7387	0.5518
A_3	-0.7320	0.3470
A_4	-0.8113	0.4757
A_5	-0.6872	0.5974

Step 4: The outcome of the computation of the score values indicates that the expert-defined criteria are better matched by the electric vehicle A_1.

6.2. Influence of the Parameter **ς** *on Decision Outcomes*

Using the T-SFAAWA and T-SFAAWAG operators' recommended approaches, we establish various parametric values of ς. By taking into account various parametric values, we investigated how our present methodologies affected the outcomes. Table 4 and Table 5 contain a list of every investigated outcome from the T-SFAAWA and T-SFAAWG operators. It is evident from the findings that A_1 is the best.

Table 4
Results of the T-SFAAWA operator for different ς values

	$Score(A_1)$	$Score(A_2)$	$Score(A_3)$	$Score(A_4)$	$Score(A_5)$	$Ranking$
$\varsigma = 1$	-0.1594	-0.2627	-0.2745	-0.2970	-0.2332	$A_1 \succ A_5 \succ A_2 \succ A_3 \succ A_4$
$\varsigma = 2$	-0.3346	-0.5292	-0.5303	-0.5852	-0.4818	$A_1 \succ A_4 \succ A_2 \succ A_3 \succ A_4$
$\varsigma = 3$	-0.4708	-0.7387	-0.7320	-0.8113	-0.6872	$A_1 \succ A_5 \succ A_3 \succ A_2 \succ A_4$
$\varsigma = 4$	-0.5783	-0.9031	-0.8983	-0.9877	-0.8561	$A_1 \succ A_5 \succ A_3 \succ A_2 \succ A_5$

Table 5
Results of the T-SFAAWG operator for different ς values

	$Score(A_1)$	$Score(A_2)$	$Score(A_3)$	$Score(A_4)$	$Score(A_5)$	$Ranking$
$\varsigma = 1$	-0.1836	-0.3394	-0.4586	-0.3934	-0.2817	$A_1 \succ A_5 \succ A_2 \succ A_4 \succ A_3$
$\varsigma = 2$	0.4086	0.2269	0.0309	0.1501	0.2840	$A_1 \succ A_5 \succ A_2 \succ A_4 \succ A_3$
$\varsigma = 3$	0.7040	0.5518	0.340	0.4757	0.5974	$A_1 \succ A_5 \succ A_2 \succ A_4 \succ A_3$
$\varsigma = 4$	0.8491	0.7371	0.5519	0.6725	0.7715	$A_1 \succ A_5 \succ A_2 \succ A_4 \succ A_3$

6.3. Comparative Study

Sarfraz *et al.* [39] established the T-SF prioritized weighted AOs. Huang [53] presented the T-SF average and geometric operators based on power-weighted aggregation tools. Zhang *et al.* [54] introduced the geometric operators based on several important degrees of characteristics and the T-SF average. Hussain *et al.* [55] gave the idea of unknown weights in interval-value T-SF information. Ali *et al.* [56] developed a multi-criteria decision-making approach on complex T-FS Aczel-Alsina AO.

To ascertain the reliability and compatibility of the T-SFAWA and T-SFAWG operators, we used certain pre-existing methodologies to analyze their possibilities. The results of the score values and their ordering are shown in Table 6. The results of all operators are the same; i.e. A_1 is a high score value in different operators.

Table 6
Comparison results

Operator	Ranking
$\mathrm{T-SFAAWA}$	$A_1 \succ A_3 \succ A_2 \succ A_5 \succ A_4$
$\mathrm{T-SFAAWG}$	$A_1 \succ A_5 \succ A_2 \succ A_4 \succ A_3$
Sarfraz $et\ al.$ [39]	$A_1 \succ A_3 \succ A_2 \succ A_5 \succ A_4$
Sarfraz $et\ al.$ [39]	$A_3 \succ A_5 \succ A_2 \succ A_4 \succ A_1$
Huang $et\ al.$ [53]	$A_1 \succ A_3 \succ A_2 \succ A_5 \succ A_4$
Huang $et\ al.$ [53]	$A_3 \succ A_5 \succ A_2 \succ A_4 \succ A_1$
Zhang $et\ al.$ [54]	$A_1 \succ A_3 \succ A_2 \succ A_5 \succ A_4$
Zhang $et\ al.$ [54]	$A_1 \succ A_5 \succ A_2 \succ A_4 \succ A_3$
Abrar Hussain $et\ al.$ [49]	$A_1 \succ A_3 \succ A_2 \succ A_5 \succ A_4$
Abrar Hussain $et\ al.$ [49]	$A_1 \succ A_5 \succ A_2 \succ A_4 \succ A_3$
Jawad Ali $et\ al.$ [50]	$A_1 \succ A_3 \succ A_2 \succ A_5 \succ A_4$
Jawad Ali $et\ al.$ [50]	$A_1 \succ A_5 \succ A_2 \succ A_4 \succ A_3$

6. Conclusions

We presented some fundamental T-SF operations in this article, based on the *t*-conorm and *t*-norm of Aczel-Alsina. The Aczel-Alsina AOs were then given the task of aggregating the T-SF data. We demonstrate several intriguing characteristics of these AOs, such as boundedness, idempotency, and monotonicity.

It is essential to assess our current hypotheses using T-SFS data when experts are unable to produce an optimal or useful solution. In order to deal with difficult situations, we need to expand the scope of our proposed research in the context of T-SFS and the intricate T-SFS circumstances theory. Following that, we will apply our distinctive methods to issues in a range of real-world sectors, including game theory, computation, web development, artificial intelligence, waste management, pattern recognition, logistics operators, social sciences, and networking. Future developments would make it possible for it to assist in solving difficult practical issues like multi-party decision-making in the governance of urban transportation.

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Conflicts of Interest
The author declares no conflicts of interest.

Appendix 1: Proof of Theorem 1

We can establish Theorem 1 with the aid of the mathematical induction technique in the following way: For $\vartheta = 2$, using Aczel-Alsina operations of T-SFVs, we obtain:

$$\boldsymbol{K_1\beta_1} = \left(\sqrt[\mathbb{T}]{\mathbf{1}-e^{-\left(K_j\left(-log\left(1-s_1^{\mathbb{T}}\right)\right)^{\varsigma}\right)^{1/\varsigma}}}, e^{-\left(K_j\left(-log(1-i_1)\right)^{\varsigma}\right)^{\frac{1}{\varsigma}}}, e^{-\left(K_j\left(-log(1-d'_1)\right)^{\varsigma}\right)^{1/\varsigma}}\right),$$

$$\boldsymbol{K_2\beta_2} = \left(\sqrt[\mathbb{T}]{\mathbf{1}-e^{-\left(K_j\left(-log\left(1-s_2^{\mathbb{T}}\right)\right)^{\varsigma}\right)^{1/\varsigma}}}, e^{-\left(K_j\left(-log(1-i_2)\right)^{\varsigma}\right)^{\frac{1}{\varsigma}}}, e^{-\left(K_j\left(-log(1-d'_2)\right)^{\varsigma}\right)^{\frac{1}{\varsigma}}}\right).$$

$$\boldsymbol{K_1\beta_1 \oplus K_2\beta_2} = \left(\sqrt[\mathbb{T}]{\mathbf{1}-e^{-\left(K_j\left(-log\left(1-s_1^{\mathbb{T}}\right)\right)^{\varsigma}\right)^{1/\varsigma}}}, e^{-\left(K_j\left(-log(1-i_1)\right)^{\varsigma}\right)^{\frac{1}{\varsigma}}}, e^{-\left(K_j\left(-log(1-d'_1)\right)^{\varsigma}\right)^{1/\varsigma}}\right)$$

$$\oplus\left(\sqrt[\mathbb{T}]{\mathbf{1}-e^{-\left(K_j\left(-log\left(1-s_2^{\mathbb{T}}\right)\right)^{\varsigma}\right)^{1/\varsigma}}}, e^{-\left(K_j\left(-log(1-i_2)\right)^{\varsigma}\right)^{\frac{1}{\varsigma}}}, e^{-\left(K_j\left(-log(1-d'_2)\right)^{\varsigma}\right)^{\frac{1}{\varsigma}}}\right)$$

$$= \left(\begin{array}{c}\sqrt[\mathbb{T}]{\mathbf{1}-e^{-\left(\left(K_j\left(-log\left(1-s_1^{\mathbb{T}}\right)\right)^{\varsigma}\right)+\left(K_j\left(-log\left(1-s_2^{\mathbb{T}}\right)\right)^{\varsigma}\right)\right)^{\frac{1}{\varsigma}}}}, e^{-\left(\left(K_j\left(-log(1-i_1)\right)^{\varsigma}\right)+\left(K_j\left(-log(1-i_2)\right)^{\varsigma}\right)\right)^{\frac{1}{\varsigma}}}, \\ e^{-\left(\left(K_j\left(-log(1-d'_1)\right)^{\varsigma}\right)+\left(K_j\left(-log(1-d'_2)\right)^{\varsigma}\right)\right)^{1/\varsigma}}\end{array}\right)$$

$$= \left(\sqrt[\mathbb{T}]{\mathbf{1}-e^{-\left(\sum_{j=1}^{2}\left(K_j\left(-log\left(1-s_j^{\mathbb{T}}\right)\right)^{\varsigma}\right)\right)^{1/\varsigma}}}, e^{-\left(\sum_{j=1}^{2}\left(K_j\left(-log(1-i_j)\right)^{\varsigma}\right)\right)^{1/\varsigma}}, e^{-\left(\sum_{j=1}^{2}\left(K_j\left(-log(1-d'_j)\right)^{\varsigma}\right)\right)^{1/\varsigma}}\right).$$

Hence, it is true for $\vartheta = 2$.

Assume that it is true for $\vartheta = a$. Then, we have:

$$\boldsymbol{T-SFAAWA(\beta_1,\beta_2,\ldots\beta_a) = \oplus_{j=1}^{a} K_j\beta_j = \oplus_{j=1}^{a} K_j\beta_j}$$

$$= \left(\sqrt[\mathbb{T}]{\mathbf{1}-e^{-\left(\sum_{j=1}^{a}\left(K_j\left(-log\left(1-s_j^{\mathbb{T}}\right)\right)^{\varsigma}\right)\right)^{1/\varsigma}}}, e^{-\left(\sum_{j=1}^{a}\left(K_j\left(-log(1-i_j)\right)^{\varsigma}\right)\right)^{1/\varsigma}}, e^{-\left(\sum_{j=1}^{a}\left(K_j\left(-log(1-d'_j)\right)^{\varsigma}\right)\right)^{1/\varsigma}}\right).$$

Now for $\vartheta = a+1$, we get:

$$\boldsymbol{T-SFAAWA(\beta_1,\beta_2,\ldots\beta_a,\beta_{a+1}) = \oplus_{j=1}^{a+1} K_j\beta_j = \oplus_{j=1}^{a} K_j\beta_j \oplus K_{a+1}\beta_{a+1}}$$

$$= \left(\begin{array}{c}\left(\sqrt[\mathbb{T}]{\mathbf{1}-e^{-\left(\sum_{j=1}^{a}\left(K_j\left(-log\left(1-s_j^{\mathbb{T}}\right)\right)^{\varsigma}\right)\right)^{1/\varsigma}}}, e^{-\left(\sum_{j=1}^{a}\left(K_j\left(-log(1-i_j)\right)^{\varsigma}\right)\right)^{1/\varsigma}}, e^{-\left(\sum_{j=1}^{a}\left(K_j\left(-log(1-d'_j)\right)^{\varsigma}\right)\right)^{1/\varsigma}}\right) \\ \oplus\left(\sqrt[\mathbb{T}]{\mathbf{1}-e^{-\left(K_{a+1}\left(-log\left(1-s_{a+1}^{\mathbb{T}}\right)\right)^{\varsigma}\right)^{1/\varsigma}}}, e^{-\left(K_{a+1}\left(-log(1-i_{a+1})\right)^{\varsigma}\right)^{1/\varsigma}}, e^{-\left(K_{a+1}\left(-log(1-d'_{a+1})\right)^{\varsigma}\right)^{1/\varsigma}}\right)\end{array}\right)$$

$$= \left(\begin{array}{c}\sqrt[\mathbb{T}]{\mathbf{1}-e^{-\left(\sum_{j=1}^{a+1}\left(K_j\left(-log\left(1-s_1^{\mathbb{T}}\right)\right)^{\varsigma}\right)\right)^{\frac{1}{\varsigma}}}}, \\ e^{-\left(\sum_{j=1}^{a+1}\left(K_j\left(-log(i_j)\right)^{\varsigma}\right)\right)^{1/\varsigma}}, e^{-\left(K_j\left(\sum_{j=1}^{a+1}\left(-log(d'_j)\right)^{\varsigma}\right)\right)^{1/\varsigma}}\end{array}\right).$$

So, after investigating, we determine that the aforementioned equation is true for $= a+1$.

Therefore, we have:

$$T - SFAAWA = \begin{pmatrix} \sqrt[\mathbb{T}]{1 - e^{-\left(\sum_{j=1}^{\vartheta}\left(K_j\left(-log\left(1-s_1^{\mathbb{T}}\right)\right)^{\varsigma}\right)\right)^{\frac{1}{\varsigma}}}}, \\ e^{-\left(\sum_{j=1}^{\vartheta}\left(K_j\left(-log(i_j)\right)^{\varsigma}\right)\right)^{\frac{1}{\varsigma}}}, e^{-\left(K_j\left(\sum_{j=1}^{\vartheta}\left(-log(d'_j)\right)^{\varsigma}\right)\right)^{\frac{1}{\varsigma}}} \end{pmatrix}.$$

Appendix 2: Proof of Theorem 2

Since $\beta_j = (s_j, i_j, d'_j)$, then we have:

$$T - SFAAWA(\beta_1, \beta_2, \dots \beta_\vartheta) = \begin{pmatrix} \sqrt[\mathbb{T}]{1 - e^{-\left(\sum_{j=1}^{\vartheta}\left(K_j\left(-log\left(1-s_1^{\mathbb{T}}\right)\right)^{\varsigma}\right)\right)^{\frac{1}{\varsigma}}}}, \\ e^{-\left(\sum_{j=1}^{\vartheta}\left(K_j\left(-log(i_j)\right)^{\varsigma}\right)\right)^{1/\varsigma}}, e^{-\left(K_j\left(\sum_{j=1}^{\vartheta}\left(-log(d'_j)\right)^{\varsigma}\right)\right)^{1/\varsigma}} \end{pmatrix}$$

$$= \begin{pmatrix} \sqrt[\mathbb{T}]{1 - e^{-\left(\sum_{j=1}^{\vartheta}\left(K_j\left(-log\left(1-s^{\mathbb{T}}\right)\right)^{\varsigma}\right)\right)^{\frac{1}{\varsigma}}}}, \\ e^{-\left(\sum_{j=1}^{\vartheta}\left(K_j\left(-log(i)\right)^{\varsigma}\right)\right)^{1/\varsigma}}, e^{-\left(K_j\left(\sum_{j=1}^{\vartheta}\left(-log(d')\right)^{\varsigma}\right)\right)^{1/\varsigma}} \end{pmatrix} = (\boldsymbol{s}, \boldsymbol{i}, \boldsymbol{d}') = \boldsymbol{\beta}.$$

Thus, $IFPAAA(\beta_1, \beta_2, \dots, \beta_\varrho) = \beta$ holds.

Appendix 3: Proof of Theorem 3

Let $\beta_j = (s_j, i_j, d'_j)$ be a set of T-SFVs, and let $\beta^- = min(\beta_1, \beta_2, \dots, \beta_\vartheta) = (s_j^-, i_j^-, d'^-_j)$ and $\beta^+ = max(\beta_1, \beta_2, \dots, \beta_\vartheta) = (s_j^+, i_j^+, d'^+_j)$. There are the subsequent inequalities:

$\boldsymbol{s^- = min_j s_j, i^- = max_j i_j, d'^- = max_j d'_j}$,

and

$\boldsymbol{s^+ = max_j s_j, i^+ = min_j i_j, d'^+ = min_j d'_j}$.

Therefore, we have the following inequities:

$$\sqrt[\mathbb{T}]{1 - e^{-\left(\sum_{j=1}^{\vartheta}\left(K_j\left(-log\left(1-(s^-)^{\mathbb{T}}\right)\right)^{\varsigma}\right)\right)^{\frac{1}{\varsigma}}}} \leq \sqrt[\mathbb{T}]{1 - e^{-\left(\sum_{j=1}^{\vartheta}\left(K_j\left(-log\left(1-s^{\mathbb{T}}\right)\right)^{\varsigma}\right)\right)^{\frac{1}{\varsigma}}}} \leq \sqrt[\mathbb{T}]{1 - e^{-\left(\sum_{j=1}^{\vartheta}\left(K_j\left(-log\left(1-(s^+)^{\mathbb{T}}\right)\right)^{\varsigma}\right)\right)^{\frac{1}{\varsigma}}}},$$

$$e^{-\left(\sum_{j=1}^{\vartheta}\left(K_j\left(-log(i^-)\right)^{\varsigma}\right)\right)^{1/\varsigma}} \geq e^{-\left(\sum_{j=1}^{\vartheta}\left(K_j\left(-log(i)\right)^{\varsigma}\right)\right)^{\frac{1}{\varsigma}}} \geq e^{-\left(\sum_{j=1}^{\vartheta}\left(K_j\left(-log(i^+)\right)^{\varsigma}\right)\right)^{\frac{1}{\varsigma}}},$$

$$e^{-\left(\sum_{j=1}^{\vartheta}\left(K_j\left(-log(d'^-)\right)^{\varsigma}\right)\right)^{1/\varsigma}} \geq e^{-\left(\sum_{j=1}^{\vartheta}\left(K_j\left(-log(d')\right)^{\varsigma}\right)\right)^{\frac{1}{\varsigma}}} \geq e^{-\left(\sum_{j=1}^{\vartheta}\left(K_j\left(-log(d'^+)\right)^{\varsigma}\right)\right)^{\frac{1}{\varsigma}}}.$$

Therefore:

$\boldsymbol{\beta^- \leq T - SFAAWA(\beta_1, \beta_2, \dots, \beta_\vartheta \leq \beta^+)}$.

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